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The Entrepreneur's Cost of Capital:

Incorporating Downside Risk in the Buildup Method

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Abstract

Capital Asset Pricing Model (CAPM) suggests that an investor's cost of equity capital is determined by beta, a measure of systematic risk based on how returns co-move with the overall market. We propose to replace beta with downside beta, a measure more consistent with investors' perception of risk. Recent empirical evidence suggests that downside beta better captures the risk-return relationship in both emerging markets and developed markets. Grouping companies into industries by two-digit SIC code, we show that the average downside beta can be very different from traditional beta. The authors discuss the implications of using downside beta on valuation, and provide a simple example to illustrate the application in valuation for both diversified and un-diversified investors.

Introduction

Investors are risk-averse. That is, they try to avoid losses and would not mind if their investments produced higher than expected returns. However, modern portfolio theory measures risk in terms of standard deviation of asset returns, which treats both positive and negative deviations from expected returns as risk. The positive deviations can be labeled "upside risk" and negative deviations "downside risk". The upside risk is not necessarily undesirable. For example, if a project provided better than expected cash flows, or a stock had higher than average return, such outcome from "upside risk" would certainly be welcome by investors. On the other hand, investors would avoid downside risk. This asymmetric view on upside risk and downside risk is certainly not reflected by standard deviation.

The concept of beta in the Capital Asset Pricing Model (CAPM) faces a similar problem. Beta is a measure of systematic risk. By definition, it measures how an investment co-moves with the market, whether the market is going up (leading to profits), or going down (leading to losses). It is then plugged into an equation for the required return of an investment. This means that if an investment provided large positive returns when the market went up, and limited losses when the market went down, it would still have a high beta and thus investors should require a higher rate of return for bearing this "risk". However, this is not consistent with how investors view risk.

This paper discusses downside risk and downside beta, and how one can incorporate downside beta in cost of equity capital estimation. We begin by reviewing CAPM and the buildup method, followed by a discussion of problems with risk measurement in modern portfolio theory. We then discuss the theoretical background on downside risk and recent empirical evidence on downside beta. Empirically, we compute both downside beta and standard beta for US companies and group them into industries by two-digit SIC code. We show that two-thirds of the time, at the industry average level, standard beta would under-estimate downside beta, which would lead to value over-estimation and for the other one-third of the time, standard beta would over-estimate downside beta, leading to value under-estimation. Finally, we use a simple example to illustrate how an entrepreneur can incorporate downside beta in the cost of equity estimation.

Buildup Method and CAPM

A typical buildup model estimates the cost of equity capital by the following equation:[1]

$$E(R_i) = R_f + RP_m + RP_s + RP_u \tag{1}$$

[1] See, for example, Pratt, S.P. and Grabowski, R.J. (2010) "*Cost of Capital: Applications and Examples*", Chapter 7.

where:

$E(R_i)$ = required return on security i

R_f = risk-free rate

RP_m = general market risk premium

RP_s = risk premium for small size

RP_u = risk premium due to company-specific risk factor[2]

In this equation, the general market risk premium is the excess return of the overall market compared to the return earned on a risk-free asset.

Alternatively, for investors in large, publicly-traded firms, the cost of equity capital can be measured by CAPM, which shows that for an investor with a well-diversified portfolio, the required return on a security can be measured by the following equation:

$$E(R_i) = R_f + \beta * RP_m \quad (2)$$

where β measures the systematic risk, or the sensitivity of a security's excess return (over the risk-free rate) to the excess return on the overall market, such as the NYSE Composite Index or S&P 500 Index.

CAPM is widely used in large publicly traded corporations in estimating the cost of equity capital. For example, Graham and Harvey (2001)[3] found that 73.5% of financial executives surveyed "always or almost always use the CAPM" for estimating the cost of equity capital. Bruner et al. (1998)[4] also found that CAPM is the dominant model for estimating cost of equity capital: it is used by 80% of the corporations and 81% of the financial advisors surveyed.

Admittedly, for privately held firms, beta cannot be directly measured since the market price of equity is unobservable. However, one can estimate a proxy beta using comparable firms operating in the same industry. As suggested by Pratt and Grabowski (2010),[5] an average or median industry beta can be estimated using publicly traded firms, which is then adjusted by leverage. The leverage-adjusted beta, via Equation (2), is then employed to compute the estimated cost of equity capital.

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[2] Examples include industry risk factor, country risk factor, or other "specific risk" factors for investors who do not hold a perfectly diversified portfolio.
[3] Graham, J.R. and Harvey, C.R. (2001) "The theory and practice of corporate finance: Evidence from the field", *Journal of Financial Economics*, 60(2), pp.187-243.
[4] Bruner, R.F., Eades, K.M., Harris, R.S. and Higgins, R.C. (1998) "Best practices in estimating the cost of capital: Survey and synthesis", *Financial Practice and Education*, Spring/Summer, pp. 13-28.
[5] Pratt, S.P. and Grabowski, R.J. (2010) "*Cost of Capital: Applications and Examples*", Chapter 11.

There is evidence showing that business appraisers also use CAPM in cost of equity capital estimation. Dukes et al. (1996)[6] surveyed members of the American Society of Appraisers (ASA) and the Institute of Business Appraisers (IBA) on the valuation practice of closely-held firms. They found that when estimating the discount rate, the first choice for practitioners was to use the risk-free rate plus a risk premium. Furthermore, the most popular method to estimate the risk premium was to multiply a market risk premium by the beta of the firm, where beta is derived from a group of publicly-traded comparable firms.

Additionally, CAPM can be augmented to incorporate size premium and specific risk. This is particularly important for investors in privately-held firms who often do not hold a well-diversified portfolio. The resulted equation is very similar to the buildup model in Equation (1), with the market risk premium replaced by the product of beta times the market risk premium:

$$E(R_i) = R_f + \beta * RP_m + RP_s + RP_u \quad (3)$$

Modern Portfolio Theory: Problems

Modern portfolio theory considers risk of an asset in two different settings: if an asset is held in isolation, then the *total risk* measured by *standard deviation* is the relevant measure of risk; if an asset is part of a well-diversified portfolio, then the *systematic risk* is the relevant measure of risk, which is measured by *beta*.

Standard deviation measures how returns deviate from its mean. Both positive and negative deviation will contribute equally to the magnitude of standard deviation. However, as discussed previously, investors would treat upside risk differently from downside risk.

Similarly, the measure of systematic risk (beta) faces the same problem. Beta measures how an asset return co-moves with the market. Consider two hypothetical stocks: U and L. Suppose the returns on U, L and the market follow the probability distribution below:

Table 1: Returns on the Market, Stocks U and L.

Probability	**Market**	**U**	**L**
0.25	-20%	-10%	-40%
0.25	-10%	-5%	-20%
0.25	10%	20%	5%
0.25	20%	40%	10%

[6] Dukes, W.P., Bowlin, O.D. and Ma, C.K. (1996) "Valuation of closely-held firms: A survey", *Journal of Business Finance and Accounting* 23(3), pp.419-38.

The distribution table shows that, when the market goes down, U goes down half as the market while L goes down twice as the market. And when the market goes up, U goes up twice as the market while L goes up only half as the market. Most investors would probably agree L is riskier than U, and therefore require a higher return to hold L. However, according to CAPM, U and L would have the same beta, and thus investors would require the same rate of return.

This example highlights the important difference between investor's perception of risk and the measure of beta. Investors focus on the downside risk, while beta measures both upside and downside risk. Below, we discuss the measure of downside risk, its theoretical background, and recent empirical evidence.

Downside Risk

The measure of downside risk was developed at about the same time that Markowitz developed the mean-variance theory.[7] Roy (1952)[8] was the first to formally model downside risk. In his model, an investor would prefer safety of principal first, and the resulting technique is termed "safety-first" technique. The measure focuses on the downside loss, rather than on the entire distribution of returns.

Generally speaking, downside risk can be measured relative to the mean, or to any benchmark return. The resulting statistic is known as semi-variance, whose square root is termed semi-deviation.

Denote semi-deviation with respect to a benchmark B as $\sum_B$, we have:

$$\textstyle\sum_B = \sqrt{\left(\frac{1}{T}\right) * \sum_{t=1}^{T} [\min(R_t - B), 0]^2} \quad (4)$$

When B is the mean return, semi-deviation measures the spread of outcomes below the mean. However, semi-deviation can also be measured to any benchmark returns, thus making it an effective tool in risk analysis. In fact, this measure of downside risk has been widely used in the investment community to evaluate fund performance.[9]

As can be seen from the formula, only returns below a certain threshold would be considered in semi-deviation. Thus, the focus is on the downside of asset returns. This is consistent with investors' perception of risk. In fact, Markowitz himself admitted that "semi-variance is the more

[7] Markowitz, H. (1952) "The utility of wealth", *Journal of Political Economy* 60(2), pp.151-58; Markowitz, H. (1952) "Portfolio selection", *Journal of Finance* 7(1), pp.77-91.
[8] Roy, A.D. (1952) "Safety first and the holding of assets", *Econometrica* 20(3), pp. 431-49.
[9] See for example, Sortino, F.A. and Meer, R.V. (1991) "Downside Risk", *Journal of Portfolio Management* 17(4), pp.27-31; Sortino, F.A., and Price, L.N. (1994) "Performance measurement in a downside risk framework", *Journal of Investing* 3(3), pp.59-64; Mamoghli C. and Daboussi S. (2007) "Performance Measurement of Hedge Funds Portfolios in a Downside Risk Framework", *Journal of Wealth Management* 12(2), pp.101-12.

plausible measure of risk."[10] However, he chose to use the mean-variance framework for computational reasons.

Another reason for using the downside risk measure is that security distributions may not be normal or symmetrical. Markowitz (1959)[11] shows that when distributions are normal, both the semi-variance and the variance measure are equivalent. Intuitively, since the normal distribution is symmetrical, the upper-tail and lower-tail would provide the same information. Thus, semi-variance would simply be one-half of variance. However, empirical research has shown that security distributions are non-normal, and non-symmetrical.[12] Therefore, focusing on the part of the distribution below the threshold value gives a more accurate portrait of downside risk.

Empirically, researchers have shown that this measure of downside risk is more consistent with investors' perception of risk. For example, in a survey of business executives in medium and large companies, when asked what they understood by the term "investment risk", the typical answers were:

> "*Risk is the prospect of not meeting the target rate of return. That is the risk, isn't it? If you are one hundred percent sure of making the target return, then it is a zero risk proposition.*"
>
> "*…I never worry about the project return going above the target return. Risk is what might happen when the return is going to be less.*"[13]

These statements clearly show that investors view risk as the probability of loss. Consistent with this view, investors would require higher return to hold assets with higher downside risk.

In fact, Hogan and Warren (1974)[14] showed that CAPM can be revised with semi-standard deviation in place of standard deviation to measure portfolio risk. Bawa and Lindenberg (1977)[15] demonstrated that CAPM can be naturally extended to incorporate the measure of downside beta as below:

$$\beta^{-} = \frac{cov(r_i, r_m | r_m < u_m)}{var(r_m | r_m < u_m)} \quad (5)$$

[10] Markowitz, H. (1991) "Portfolio selection: Efficient diversification of investment (2e)", Blackwell Publishers Inc., Malden, MA.
[11] Markowitz, H. (1959) "Portfolio Selection (1e)", John Wiley & Sons, New York.
[12] Mandelbrot , B. (1963) "The variation of certain speculative prices", *Journal of Business* 36(4), pp.394-419; Bekaert G., Erb, C., Harvey, C., and Viskanta, T.(1998) "Distributional characteristics of emerging market returns, and asset allocation", *Journal of Portfolio Management* 24, pp.102-16; Aparicio, F. and Estrada, J. (2001) "Empirical distributions of stock returns: European securities markets, 1990-95", *European Journal of Finance* 7, pp. 1-21.
[13] Mao, J.C.T. (1970), "Survey of capital budgeting: Theory and practice", *Journal of Finance* 25(2), pp. 349-60.
[14] Hogan, W.W. and Warren, J.M. (1974) "Toward the development of an equilibrium capital-market model based on semi-variance", *Journal of Financial and Quantitative Analysis* 9(1), pp. 1-11.
[15] Bawa, V. and Lindenberg, E. (1977) "Capital market equilibrium in a mean-lower partial moment framework", *Journal of Financial Economics* 5, pp. 189-200.

where r_i and r_m are the excess returns to security i and market m, and u_m is the average market excess return.

Similarly, an upside beta can be measured as follows:

$$\beta^{+} = \frac{cov(r_i, r_m | r_m > u_m)}{var(r_m | r_m > u_m)} \tag{6}$$

Therefore, β^- and β^+ can be estimated with a regression of excess return of security i on excess return of the market, conditional on excess market return being below the mean (downside beta) and above the mean (upside beta).

Recent empirical studies have shown increasing support on using downside beta. In the US market, Ang et al. (2006)[16] showed that investors are rewarded with higher returns for bearing downside risks. The authors estimated that the cross-section of stock returns exhibits a downside risk premium of about 6% annually. More importantly, they showed that the reward for bearing downside risk cannot be explained by standard market beta, size, or value, nor can it be explained by liquidity risk or momentum characteristics. They also illustrated that individual stocks with higher downside betas have higher average returns over the same period.

Besides, downside beta has been shown to explain the cross-section returns better than standard beta among other developed markets.[17] In emerging markets, Alles and Murray (2013)[18] studied eight Asian markets[19] and found that downside beta is priced by investors.

A series of studies by Estrada[20] supported the use of downside beta in both emerging markets and developed markets. In a comprehensive study,[21] he computed standard betas and downside betas for stocks across 23 developed markets and 27 emerging markets included in the Morgan Stanley Capital Indices (MSCI). He found that downside beta outperformed standard beta in explaining the variations of cross-section returns in both types of market. In particular, across emerging market companies, downside beta alone explained 55% of the variability in mean returns.

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[16] Ang, A, Chen, J. and Xing, Y. (2006) "Downside risk", *Review of Financial Studies* 19(4), pp. 1191-1239.
[17] For an analysis on U.K. individual stocks, see: Pedersen, C. and Huang, S. (2007) "Does downside beta matter in asset pricing?" *Applied Financial Economics* 17, pp. 961-78; For an application on stocks listed on the London Stock Exchange and Paris Stock Exchange, see: Artavanis, N., Diacogiannis, G. and Mylonakis, J. (2010) "The D-CAPM: The case of Great Britain and France", *International Journal of Economics and Finance* 2(3), pp. 25-38.
[18] Alles, L., and Murray, L. (2013) "Rewards for downside risk in Asian markets", *Journal of Banking and Finance*, forthcoming.
[19] The eight Asian markets are: China, India, Indonesia, Korea, Malaysia, Pakistan, Taiwan and Thailand.
[20] Estrada, J. (2000) "The cost of equity in emerging markets: A downside risk approach", *Emerging Markets Quarterly*, Fall, pp. 19-30; Estrada, J. (2002) "Systematic risk in emerging markets: The D-CAPM", *Emerging Markets Review* 3(4), pp. 365-79; and Estrada, J. (2007) "Mean-semivariance behavior: Downside risk and capital asset pricing", *International Review of Economics and Finance*, 16, 169-85.
[21] Estrada (2007) "Mean-semivariance behavior: Downside risk and capital asset pricing", *International Review of Economics and Finance*, 16, pp. 169-85.

Estrada also showed that standard beta and downside beta could differ significantly, especially in emerging markets, where stock returns are more skewed than those in developed markets. For example, average standard beta for Brazilian firms is 1.44, while downside beta is 2.16; Indian firms' average standard beta is 0.54 while downside beta is 1.10.

Furthermore, Estrada computed the required return on equity based on both measures of beta. Using downside beta, the mean required annual return[22] of firms across developed market is 74 basis points higher than that measured by standard beta. However, in emerging markets, required return based on downside beta is 254 basis points higher than the standard beta counterpart. Such a big difference will no doubt affect the valuation of companies significantly, private or public. As the author argued in the paper, "*Differences of this magnitude are simply too large for practitioners to ignore*" (Estrada 2007, pp184).

More recently, Chong and Phillips (2012) [23]discussed implications of using downside beta on corporate valuation. They showed that even at the industry level, the average downside beta can be quite different from that of the standard beta. About half of the time the average downside beta is higher than the standard beta. Under these occasions, a financial analyst using CAPM betas would under-estimate the cost of capital and over-estimate the project or corporate value.

<u>**The Dual-beta Model and the Data**</u>

To illustrate the difference in standard beta and downside beta, we follow the model used in Chong and Phillips (2012). We estimate the following regression using daily stock returns:

$$(r_j - r_f)_t = \alpha_j^+ D + \beta_j^+ \left(r_m^+ - r_f\right)_t D + \alpha_j^- (1 - D) + \beta_j^- \left(r_m^- - r_f\right)_t (1 - D) + \varepsilon_t \qquad (7)$$

In this equation, D is a dummy variable that takes a value of 1 when the daily market return is non-negative. Thus, two sets of intercept and beta are estimated separately: α_j^- and β_j^- during market downturns (when market return is negative), and α_j^+ and β_j^+ when the market went up. We label β_j^- as "down-market beta" and β_j^+ as "up-market beta", to be precise.[24]

We compute down-market and up-market beta for 4,500 stocks traded on NYSE, NASDAQ and AMEX, using daily returns over the past year. We also compute the standard beta using the same

[22] The author used a risk-free rate of 5.03% and market risk premium of 5.5%.
[23] Chong, J. and Phillips, G.M. (2012) "Measuring risk for cost of capital: The downside beta approach", *Journal of Corporate Treasury Management* 4(4), pp. 344-52.
[24] Notice here the down-market and up-market betas are slightly different from those estimated in equation (5) and (6). The downside beta in equation (5) is estimated conditional on when the market excess return is lower than the mean excess return. Similarly, the upside beta in equation (6) is conditional on when the market excess return is above the mean excess return.

data.[25] We then group the companies into industries by two-digit SIC code, and compute the average and median standard beta and downside beta by industry.

Table 2 presents the results. As shown from the calculated difference, the down-market beta seldom equals the standard beta at the industry level. Roughly two-thirds of the time the down-market beta is higher than the standard beta. In some cases, the difference can be quite significant. Take for example, the "agricultural production – livestock and animal specialties" industry (SIC=2): the average standard beta is 0.386 while the downside beta is 1.002, more than twice the estimate on standard beta. In the cases when down-market beta is higher than the standard beta, the down-market beta is higher by about 21.3% on average.

We also computed equity rates using both down-market and standard beta. We use a risk-free rate of 2%, which is close to the 10-year Treasury note yield as of end of February 2013, and equity risk premium of 6%, as suggested in Ang et al. (2006). Thus, the equity rates are computed using:

$$k_{is} = 2\% + \beta_{is} * 6\% \tag{8}$$

where i stands for the industry and s refers to the style of beta used (down-market or standard beta).

The equity rates are presented in Table 3. In addition, assuming a hypothetical annual perpetual cash flow of $100,000, we compute the value of the cash flows using both equity rates. PV(std) shows the present value of cash flows using equity rates with standard beta, while PV(down) shows the value using equity rates with down-market beta. The last column shows the value difference. Since about two-thirds of the time the standard beta is lower than the down-market beta, using equity rates based on standard beta would over-estimate the value of the cash flows about two-thirds of the time. The average over-valuation is about 14%. For the other one-third of the time, standard beta is higher than down-market beta. This leads to value under-estimation by 7.3% on average.

Below we use a simple example to illustrate the valuation impact to an entrepreneur. For simplification, we assume the entrepreneur does not have any debt, and the annual projected cash flow is $100,000 for 10 years. The entrepreneur uses the buildup model to estimate the cost of capital. The comparable industry is the *miscellaneous repair service* with SIC=76. We choose this industry because it gives an example where downside beta is significantly higher than the standard beta.

Table 4 shows the cost of equity capital and present value calculation. We assume the size premium is 6.36%.[26] For a diversified investor, we assume there is no additional risk premium

[25] Parameter estimates were provided by MacroRisk Analytics, a commercial provider of these estimates, from their database as of February 22, 2012.

[26] Ibbotson SSBI 2011 Valuation Yearbook, Decile 10.

(RP_u=0 in Equation (1)). Based on downside beta, the estimated present value is $388,480. If the entrepreneur used the standard beta instead, he would have estimated the present value to be $516,642, leading to an over-estimation of $128,162, or about a 33% value over-estimation.

If the entrepreneur is un-diversified, the required return would be higher to reflect the investor's lack of diversification.[27] For illustration purposes, we add a risk premium of 5% for the owner's lack-of-diversification. The cost of equity capital based on downside beta and standard beta is 27.3% and 19.5% respectively. The present value based on downside beta is $333,486, while that based on standard beta is $480,212. Again, using standard beta would over-estimate value by about 30%.

Admittedly, not all industries experience such a big difference between standard beta and downside beta. However, for most of the industries the difference is still quite significant. Thus, using downside beta could potentially lead to more accurate valuation estimation. Besides, the estimation procedure on downside beta requires no additional information beyond that required by standard beta. At a minimum, it would provide an alternative, and arguably better valuation measure than those derived from standard beta.

Conclusion

There is now increasing evidence showing that downside risk is priced by investors. Downside risk focuses on the risk of losses, instead of the entire distribution of investment returns. Theoretically, this measure is more consistent with investors' perception of risk. Empirically, studies have shown that investment with higher downside risk is rewarded with higher returns, both in emerging markets and in developed markets. The reward for bearing downside risk is significant, and cannot be explained by the standard beta.

This paper reviewed the theoretical and empirical literature on downside beta. More importantly, using daily returns of about 4,500 stocks over the last year, we showed that downside beta can be quite different from standard beta, even at the industry average level. More specifically, we find that around two-thirds of the time standard beta would under-estimate the downside risk. Under these scenarios, valuation based on standard beta would lead to over-estimation.

Using a simple example, the paper illustrated how an entrepreneur could incorporate downside beta in the cost of equity capital estimation, and how using standard beta would lead to significant value over-estimation when downside beta is much higher than standard beta.

[27] For a more detailed discussion on cost of capital adjustment to reflect investors' lack of diversification, see: McConaughy, D. and Covrig, V. (2007), "Owner's lack of diversification and the cost of equity capital for a closely-held firm", *Business Valuation Review* 26(4), pp. 115-20.

Downside beta is readily available through commercial data providers or by using a spreadsheet. At a minimum, valuation based on downside beta would provide an alternative measure to the entrepreneur, perhaps leading to a more accurate valuation of the business.

Table 2: Standard Beta and Downside Beta by Two-Digit SIC Code

2 Digit SIC	Label	Standard Beta (Mean)	Down-Market Beta (Mean)	Difference (Mean)	Standard Beta (Median)	Down-Market Beta (Median)	Difference (Median)
1	Agricultural Production - Crops	0.873	0.692	0.181	1.000	0.686	0.315
2	Agricultural Production - Livestock and Animal Specialties	0.386	1.002	-0.617	0.476	0.618	-0.142
7	Agricultural Services	0.830	1.039	-0.210	0.830	1.039	-0.210
8	Forestry	0.450	0.479	-0.029	0.450	0.479	-0.029
10	Metal Mining	1.263	0.485	0.778	1.232	0.293	0.940
12	Coal Mining	1.464	1.500	-0.036	1.549	1.419	0.130
13	Oil and Gas Extraction	1.439	1.371	0.068	1.467	1.387	0.080
14	Mining and Quarrying of Nonmetallic Minerals, Except Fuels	1.284	0.856	0.428	1.321	0.807	0.513
15	Building Cnstrctn - General Contractors & Operative Builders	1.430	1.417	0.013	1.599	1.524	0.075
16	Heavy Cnstrctn, Except Building Construction - Contractors	1.243	1.364	-0.121	1.341	1.449	-0.108
17	Construction - Special Trade Contractors	1.281	1.356	-0.075	1.100	1.173	-0.072
20	Food and Kindred Products	0.743	0.747	-0.003	0.689	0.704	-0.015
21	Tobacco Products	0.751	0.874	-0.123	0.547	0.604	-0.057
22	Textile Mill Products	0.976	1.005	-0.029	1.108	1.075	0.033
23	Apparel, Finished Prdcts from Fabrics & Similar Materials	1.097	1.288	-0.191	1.150	1.335	-0.185
24	Lumber and Wood Products, Except Furniture	1.691	1.569	0.121	1.546	1.458	0.088
25	Furniture and Fixtures	1.009	1.189	-0.180	1.157	1.284	-0.127
26	Paper and Allied Products	1.083	1.100	-0.018	1.047	1.172	-0.126
27	Printing, Publishing and Allied Industries	0.956	0.987	-0.030	1.028	1.006	0.022
28	Chemicals and Allied Products	0.960	1.088	-0.128	1.009	1.103	-0.094
29	Petroleum Refining and Related Industries	1.368	1.359	0.009	1.188	1.133	0.055
30	Rubber and Miscellaneous Plastic Products	1.006	0.999	0.007	1.045	1.139	-0.093
31	Leather and Leather Products	0.946	1.099	-0.153	1.101	1.068	0.033
32	Stone, Clay, Glass, and Concrete Products	1.322	1.218	0.104	1.518	1.214	0.304
33	Primary Metal Industries	1.380	1.200	0.179	1.488	1.338	0.150
34	Fabricated Metal Prdcts, Except Machinery & Transport Eqpmnt	1.026	1.138	-0.112	1.192	1.165	0.027
35	Industrial and Commercial Machinery and Computer Equipment	1.220	1.180	0.039	1.300	1.232	0.068
36	Electronic, Elctrcl Eqpmnt & Cmpnts, Excpt Computer Eqpmnt	1.138	1.049	0.089	1.195	1.053	0.142
37	Transportation Equipment	1.370	1.333	0.037	1.362	1.235	0.127
38	Mesr/Anlyz/Cntrl Instrmnts; Photo/Med/Opt Gds; Watchs/Clocks	0.917	1.040	-0.123	0.973	1.060	-0.087
39	Miscellaneous Manufacturing Industries	0.879	0.976	-0.097	0.907	0.974	-0.067
40	Railroad Transportation	0.927	0.907	0.020	1.100	0.880	0.220
41	Local, Suburban Transit & Interurbn Hgwy Passenger Transport	0.429	0.994	-0.564	0.429	0.994	-0.564
42	Motor Freight Transportation	1.160	1.270	-0.110	1.060	1.210	-0.150
44	Water Transportation	1.061	0.990	0.071	1.080	0.986	0.094
45	Transportation by Air	0.954	1.021	-0.068	0.859	0.957	-0.098
46	Pipelines, Except Natural Gas	0.572	0.596	-0.024	0.589	0.485	0.105
47	Transportation Services	0.937	0.951	-0.013	0.909	0.811	0.099
48	Communications	0.878	1.021	-0.143	0.875	1.075	-0.201
49	Electric, Gas and Sanitary Services	0.523	0.629	-0.106	0.526	0.593	-0.067
50	Wholesale Trade - Durable Goods	0.957	0.906	0.051	1.006	1.032	-0.025
51	Wholesale Trade - Nondurable Goods	0.898	0.916	-0.018	0.848	0.915	-0.067
52	Building Matrials, Hrdwr, Garden Supply & Mobile Home Dealrs	1.116	1.197	-0.081	1.016	1.213	-0.196
53	General Merchandise Stores	0.903	0.955	-0.053	0.896	0.880	0.016
54	Food Stores	0.967	0.895	0.072	0.917	0.931	-0.014
55	Automotive Dealers and Gasoline Service Stations	0.988	0.972	0.017	1.217	1.524	-0.307
56	Apparel and Accessory Stores	1.065	1.045	0.021	1.105	1.156	-0.051
57	Home Furniture, Furnishings and Equipment Stores	0.978	0.875	0.103	1.048	0.975	0.073
58	Eating and Drinking Places	0.834	0.931	-0.097	0.924	0.893	0.032
59	Miscellaneous Retail	0.909	0.941	-0.032	0.953	1.037	-0.084
60	Depository Institutions	0.620	0.719	-0.099	0.498	0.701	-0.203
61	Nondepository Credit Institutions	0.627	0.722	-0.095	0.575	0.631	-0.056
62	Security & Commodity Brokers, Dealers, Exchanges & Services	0.630	0.666	-0.035	0.757	0.835	-0.078
63	Insurance Carriers	0.792	0.877	-0.085	0.724	0.783	-0.059
64	Insurance Agents, Brokers and Service	0.834	0.740	0.094	0.741	0.785	-0.044
65	Real Estate	0.806	0.968	-0.161	0.775	0.978	-0.203
67	Holding and Other Investment Offices	0.602	0.649	-0.047	0.744	0.839	-0.095
70	Hotels, Rooming Houses, Camps, and Other Lodging Places	1.132	1.189	-0.057	1.291	1.278	0.013
72	Personal Services	1.013	1.057	-0.043	0.971	1.110	-0.140
73	Business Services	1.003	1.025	-0.022	1.036	1.043	-0.007
75	Automotive Repair, Services and Parking	1.294	1.407	-0.114	1.319	1.301	0.018
76	Miscellaneous Repair Services	0.981	2.324	-1.343	0.981	2.324	-1.343
78	Motion Pictures	0.821	0.835	-0.015	0.873	0.870	0.002
79	Amusement and Recreation Services	0.735	0.718	0.017	0.827	0.857	-0.030
80	Health Services	0.947	1.010	-0.063	0.976	1.118	-0.142
81	Legal Services	1.023	1.335	-0.312	1.023	1.335	-0.312
82	Educational Services	0.861	1.103	-0.243	0.845	1.126	-0.280
83	Social Services	0.806	-0.381	1.187	0.806	0.505	0.301
86	Membership Organizations	1.389	1.253	0.135	1.389	1.253	0.135
87	Engineering, Accounting, Research, Management & Related Svcs	0.866	1.037	-0.171	0.834	0.890	-0.056
89	Services, Not Elsewhere Classified	1.136	1.118	0.018	1.215	1.074	0.141
95	Administration of Environmental Quality and Housing Programs	1.161	2.721	-1.560	1.161	2.721	-1.560
96	Administration of Economic Programs	1.210	1.540	-0.330	1.210	1.540	-0.330
99	Nonclassifiable Establishments	0.773	0.817	-0.045	0.900	0.924	-0.023

Table 3: Equity Rates and Present Value of Cash Flows Based on Standard Beta and Down Market Beta

2 Digit SIC	Label	Standard Beta (Mean)	BaseRate (Standard beta)	Down Market Beta (Mean)	BaseRate (Down beta)	PV (Std)	PV (Down)	(PV(Std) - PV(Down))
1	Agricultural Production - Crops	0.873	7.2%	0.692	6.2%	$ 1,381,802	$ 1,625,062	$ (243,260)
2	Agricultural Production - Livestock and Animal Specialties	0.386	4.3%	1.002	8.0%	$ 2,317,940	$ 1,247,894	$ 1,070,046
7	Agricultural Services	0.830	7.0%	1.039	8.2%	$ 1,433,244	$ 1,214,413	$ 218,831
8	Forestry	0.450	4.7%	0.479	4.9%	$ 2,127,345	$ 2,051,690	$ 75,654
10	Metal Mining	1.263	9.6%	0.485	4.9%	$ 1,044,148	$ 2,036,063	$ (991,915)
12	Coal Mining	1.464	10.8%	1.500	11.0%	$ 927,207	$ 909,233	$ 17,974
13	Oil and Gas Extraction	1.439	10.6%	1.371	10.2%	$ 940,378	$ 977,916	$ (37,538)
14	Mining and Quarrying of Nonmetallic Minerals, Except Fuels	1.284	9.7%	0.856	7.1%	$ 1,030,499	$ 1,401,812	$ (371,313)
15	Building Cnstrctn - General Contractors & Operative Builders	1.430	10.6%	1.417	10.5%	$ 945,070	$ 952,144	$ (7,074)
16	Heavy Cnstrctn, Except Building Construction - Contractors	1.243	9.5%	1.364	10.2%	$ 1,057,105	$ 982,010	$ 75,095
17	Construction - Special Trade Contractors	1.281	9.7%	1.356	10.1%	$ 1,032,657	$ 986,846	$ 45,812
20	Food and Kindred Products	0.743	6.5%	0.747	6.5%	$ 1,548,045	$ 1,543,053	$ 4,992
21	Tobacco Products	0.751	6.5%	0.874	7.2%	$ 1,536,729	$ 1,380,446	$ 156,283
22	Textile Mill Products	0.976	7.9%	1.005	8.0%	$ 1,272,792	$ 1,245,607	$ 27,185
23	Apparel, Finished Prdcts from Fabrics & Similar Materials	1.097	8.6%	1.288	9.7%	$ 1,165,366	$ 1,027,970	$ 137,395
24	Lumber and Wood Products, Except Furniture	1.691	12.1%	1.569	11.4%	$ 823,450	$ 875,920	$ (52,471)
25	Furniture and Fixtures	1.009	8.1%	1.189	9.1%	$ 1,241,776	$ 1,094,704	$ 147,072
26	Paper and Allied Products	1.083	8.5%	1.100	8.6%	$ 1,176,978	$ 1,162,573	$ 14,404
27	Printing, Publishing and Allied Industries	0.956	7.7%	0.987	7.9%	$ 1,292,576	$ 1,262,717	$ 29,860
28	Chemicals and Allied Products	0.960	7.8%	1.088	8.5%	$ 1,288,427	$ 1,172,573	$ 115,854
29	Petroleum Refining and Related Industries	1.368	10.2%	1.359	10.2%	$ 979,462	$ 984,840	$ (5,378)
30	Rubber and Miscellaneous Plastic Products	1.006	8.0%	0.999	8.0%	$ 1,244,356	$ 1,250,604	$ (6,249)
31	Leather and Leather Products	0.946	7.7%	1.099	8.6%	$ 1,302,577	$ 1,163,756	$ 138,820
32	Stone, Clay, Glass, and Concrete Products	1.322	9.9%	1.218	9.3%	$ 1,007,065	$ 1,074,342	$ (67,277)
33	Primary Metal Industries	1.380	10.3%	1.200	9.2%	$ 972,862	$ 1,086,709	$ (113,848)
34	Fabricated Metal Prdcts, Except Machinery & Transport Eqpmnt	1.026	8.2%	1.138	8.8%	$ 1,226,432	$ 1,132,974	$ 93,457
35	Industrial and Commercial Machinery and Computer Equipment	1.220	9.3%	1.180	9.1%	$ 1,073,285	$ 1,101,246	$ (27,961)
36	Electronic, Elctrcl Eqpmnt & Cmpnts, Excpt Computer Eqpmnt	1.138	8.8%	1.049	8.3%	$ 1,132,839	$ 1,205,575	$ (72,736)
37	Transportation Equipment	1.370	10.2%	1.333	10.0%	$ 978,461	$ 1,000,370	$ (21,909)
38	Mesr/Anlyz/Cntrl Instrmnts; Photo/Med/Opt Gds; Watchs/Clocks	0.917	7.5%	1.040	8.2%	$ 1,333,118	$ 1,213,593	$ 119,524
39	Miscellaneous Manufacturing Industries	0.879	7.3%	0.976	7.9%	$ 1,374,484	$ 1,272,984	$ 101,500
40	Railroad Transportation	0.927	7.6%	0.907	7.4%	$ 1,322,213	$ 1,343,712	$ (21,499)
41	Local, Suburban Transit & Interurbn Hgwy Passenger Transport	0.429	4.6%	0.994	8.0%	$ 2,185,376	$ 1,255,994	$ 929,382
42	Motor Freight Transportation	1.160	9.0%	1.270	9.6%	$ 1,116,175	$ 1,039,822	$ 76,353
44	Water Transportation	1.061	8.4%	0.990	7.9%	$ 1,195,613	$ 1,259,540	$ (63,927)
45	Transportation by Air	0.954	7.7%	1.021	8.1%	$ 1,294,867	$ 1,230,278	$ 64,589
46	Pipelines, Except Natural Gas	0.572	5.4%	0.596	5.6%	$ 1,840,985	$ 1,792,958	$ 48,027

Table 3 continued

2 Digit SIC	Label	Standard Beta (Mean)	BaseRate (Standard beta)	Down Market Beta (Mean)	BaseRate (Down beta)	PV (Std)	PV (Down)	(PV(Std) - PV(Down))
47	Transportation Services	0.937	7.6%	0.951	7.7%	$ 1,311,559	$ 1,298,055	$ 13,504
48	Communications	0.878	7.3%	1.021	8.1%	$ 1,375,891	$ 1,230,688	$ 145,203
49	Electric, Gas and Sanitary Services	0.523	5.1%	0.629	5.8%	$ 1,946,198	$ 1,731,549	$ 214,648
50	Wholesale Trade - Durable Goods	0.957	7.7%	0.906	7.4%	$ 1,292,067	$ 1,345,090	$ (53,024)
51	Wholesale Trade - Nondurable Goods	0.898	7.4%	0.916	7.5%	$ 1,352,997	$ 1,333,582	$ 19,415
52	Building Matrials, Hrdwr, Garden Supply & Mobile Home Dealrs	1.116	8.7%	1.197	9.2%	$ 1,150,202	$ 1,089,120	$ 61,082
53	General Merchandise Stores	0.903	7.4%	0.955	7.7%	$ 1,348,331	$ 1,293,372	$ 54,959
54	Food Stores	0.967	7.8%	0.895	7.4%	$ 1,281,851	$ 1,356,711	$ (74,860)
55	Automotive Dealers and Gasoline Service Stations	0.988	7.9%	0.972	7.8%	$ 1,261,054	$ 1,277,161	$ (16,107)
56	Apparel and Accessory Stores	1.065	8.4%	1.045	8.3%	$ 1,191,688	$ 1,209,458	$ (17,771)
57	Home Furniture, Furnishings and Equipment Stores	0.978	7.9%	0.875	7.3%	$ 1,271,051	$ 1,379,006	$ (107,955)
58	Eating and Drinking Places	0.834	7.0%	0.931	7.6%	$ 1,428,257	$ 1,318,356	$ 109,901
59	Miscellaneous Retail	0.909	7.5%	0.941	7.6%	$ 1,341,547	$ 1,308,369	$ 33,178
60	Depository Institutions	0.620	5.7%	0.719	6.3%	$ 1,748,077	$ 1,584,223	$ 163,855
61	Nondepository Credit Institutions	0.627	5.8%	0.722	6.3%	$ 1,735,717	$ 1,578,887	$ 156,830
62	Security & Commodity Brokers, Dealers, Exchanges & Services	0.630	5.8%	0.666	6.0%	$ 1,729,787	$ 1,668,390	$ 61,397
63	Insurance Carriers	0.792	6.8%	0.877	7.3%	$ 1,481,042	$ 1,377,551	$ 103,491
64	Insurance Agents, Brokers and Service	0.834	7.0%	0.740	6.4%	$ 1,427,276	$ 1,552,399	$ (125,124)
65	Real Estate	0.806	6.8%	0.968	7.8%	$ 1,462,278	$ 1,281,165	$ 181,113
67	Holding and Other Investment Offices	0.602	5.6%	0.649	5.9%	$ 1,781,693	$ 1,697,051	$ 84,642
70	Hotels, Rooming Houses, Camps, and Other Lodging Places	1.132	8.8%	1.189	9.1%	$ 1,137,444	$ 1,094,716	$ 42,728
72	Personal Services	1.013	8.1%	1.057	8.3%	$ 1,237,498	$ 1,199,005	$ 38,493
73	Business Services	1.003	8.0%	1.025	8.1%	$ 1,247,487	$ 1,227,027	$ 20,460
75	Automotive Repair, Services and Parking	1.294	9.8%	1.407	10.4%	$ 1,024,412	$ 957,513	$ 66,899
76	Miscellaneous Repair Services	0.981	7.9%	2.324	15.9%	$ 1,268,101	$ 627,200	$ 640,901
78	Motion Pictures	0.821	6.9%	0.835	7.0%	$ 1,444,394	$ 1,425,975	$ 18,419
79	Amusement and Recreation Services	0.735	6.4%	0.718	6.3%	$ 1,559,968	$ 1,584,869	$ (24,901)
80	Health Services	0.947	7.7%	1.010	8.1%	$ 1,302,047	$ 1,240,797	$ 61,250
81	Legal Services	1.023	8.1%	1.335	10.0%	$ 1,229,111	$ 999,211	$ 229,901
82	Educational Services	0.861	7.2%	1.103	8.6%	$ 1,395,811	$ 1,160,094	$ 235,717
83	Social Services	0.806	6.8%	-0.381	-0.3%	$ 1,462,779	$ (34,838,593)	$ 36,301,372
86	Membership Organizations	1.389	10.3%	1.253	9.5%	$ 967,809	$ 1,050,316	$ (82,508)
87	Engineering, Accounting, Research, Management & Related Svcs	0.866	7.2%	1.037	8.2%	$ 1,389,458	$ 1,216,283	$ 173,175
89	Services, Not Elsewhere Classified	1.136	8.8%	1.118	8.7%	$ 1,134,405	$ 1,148,598	$ (14,193)
95	Administration of Environmental Quality and Housing Programs	1.161	9.0%	2.721	18.3%	$ 1,115,683	$ 545,716	$ 569,967
96	Administration of Economic Programs	1.210	9.3%	1.540	11.2%	$ 1,079,599	$ 889,594	$ 190,005
99	Nonclassifiable Establishments	0.773	6.6%	0.817	6.9%	$ 1,507,031	$ 1,448,662	$ 58,369

Table 4: Sample Impact of Downside and Standard Beta on Valuation

Build up Model (assuming comparable industry SIC=76)		
Risk-free rate	2%	
Equity risk premium	6%	
size premium *	6.36%	
lack-of-diversification premium**	5%	
annual CF (for 10 years)	$100,000	
downside beta	2.324	
standard beta	0.981	
Diversified investor	**downside beta**	**standard beta**
cost of equity	22.30%	14.25%
Present Value	$388,480	$516,642
Value difference (standard-down)	$128,162	
Un-diversified Investor	**downside beta**	**standard beta**
cost of equity	27.30%	19.25%
Present Value	$333,486	$430,212
Value difference (standard-down)	$96,726	
*Size premium, group 10, last page of SBBI Valuation Edition 2011		
** For a more detailed discussion on diversification discount, see McConaughy and Covrig (2007).		